conditions and the assumption, in deriving Eqs. (1) and (3), that the interaction time does not exceed 10 min and the shapes and sizes of the bodies experience negligible change. In the experimental determination of local mass-transfer coefficients the samples are washed for an hour, and the shapes and sizes of the bodies are changed appreciably.

NOTATION

u, flow velocity; τ, interaction time; k_x, local mass-transfer coefficient; Δh_x, decrease in linear dimension of sample; ρ, density of test sample; c_s, saturation concentration; θ, angular distance from front critical point; r, distance from axis of sample to a point on front and rear surfaces; l, distance to a point from beginning of lateral surface; R, radius of sample; L, length of lateral surface; $Nu = kd/D$, Nusselt number; $Pr = \nu/D$, Prandtl number; $Re = ud/\nu$, Reynolds number.

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TRANSFER EQUATION IN THERMODIFFUSION COLUMN WITH SPIRAL WINDING

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The derivation of the transfer equation for a thermodiffusion column with spiral winding is outlined. It is shown that, in contrast to the classical case, this is a two-dimensional equation.

In 1962, Washal and Melpolder suggested that the separation of liquid mixtures could be intensified by winding onto the internal cylinder of the thermodiffusion column a wire spiral of diameter equal to the gap between the hot and cold surfaces of the column [1]. In their experiments with a mixture of cis and trans isomers of decahydronaphthalene, they found that for some winding angles of the spiral the degree of separation was higher by a factor of 10 than in the column without a spiral.

This result aroused the interest of researchers, and a number of papers [2-6] were devoted to the verification of the "spiral effect," since it is simple to utilize from a constructional viewpoint and allows significantly better enrichment of the mixture components to be obtained for the same energy consumption. Note, however, that the verification offered in some works [2, 5] was based not on a comparison of the results obtained with a single column with and without a spiral, but only on data obtained upon introducing into the gap a spiral with an arbitrary winding angle. In these cases, good fractionation of petroleum products and a reduction in the time of the transient process were observed. A more thorough investigation was made in [3] for binary mixtures, with different winding angles of the spiral. Only withdrawal conditions were considered; no experiments were conducted under static conditions.

The column geometry was chosen so as to satisfy the condition $c(1-c) \approx \text{const}$. It was established that there is an optimum winding angle of the spiral (depending on the rate of withdrawal and the properties of the mixture) at which the degree of separation has a clearly expressed maximum.

On the other hand, in [4-6] the introduction of a spiral winding in the working gap was not found to have any pronounced effect on the separation of the mixture. For the separation of bromium isotopes in butyl bromide [4], an increase in the time of the transient process was noted, with no increase in the degree of separation. (The experiments were conducted in the absence of withdrawal.)

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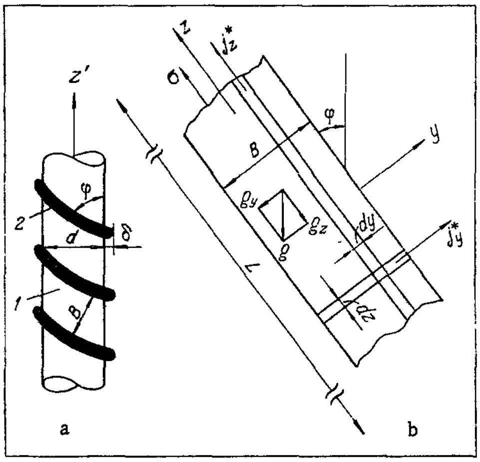

Fig. 1. Thermodiffusion column with spiral winding: 1) internal cylinder of column; 2) spiral.

In experiments on the fractionation of petroleum oil [6], both with withdrawal of material and under static conditions, using different winding angles of the spiral, it was established that at certain winding angles the time of the transient process is much reduced, but in the range of outputs investigated no maximum in the degree of separation was observed with change in the winding angle.

Unfortunately, the contradiction between the available experimental data cannot be resolved on the basis of theory, since as yet no adequate theory exists. The only attempt at an analytic description of the separation of mixtures in a column with a spiral winding [3] rests on certain incorrect assumptions which allow the problem to be reduced to a one-dimensional model.

The present paper outlines a derivation of the transfer equation for a thermodiffusional column with a spiral winding that is free from such deficiencies. A diagram of the column is given in Fig. 1a, which shows only a part of the inner cylinder with the spiral wound on it. Taking into account that in a liquid thermodiffusion column the gap is many times smaller than the radius, the space between two adjacent turns of the winding may be regarded as a plane column inclined at an angle φ to the vertical; this simplifying assumption was suggested earlier in [2] and used in [3]. The resulting plane column is shown in Fig. 1b. In the given case, bearing in mind the essentially laminar conditions of flow, the hydrodynamics may be described by the simplified Navier–Stokes equation

$$\operatorname{div}(\eta \operatorname{grad}) \mathbf{w} = \operatorname{grad} P - \rho \mathbf{g}. \tag{1}$$

As is usual in deriving the transfer equation, it is assumed that the velocity component in the x direction (perpendicular to the plane of the figure) is $w_x = 0$. It is also assumed that the velocities w_y and w_z depend only on the coordinate x. This assumption means that their variation over y is neglected, which is acceptable if $B \gg \delta$. Then in the coordinates z–y, Eq. (1) may be replaced by the relation

$$\frac{d}{dz}\left(\eta \frac{dw_z}{dx}\right) = \frac{dP}{dz} - \rho g_z, \tag{2}$$

$$\frac{d}{dy}\left(\eta \frac{dw_y}{dx}\right) = \frac{dP}{dy} - \rho g_y. \tag{3}$$

It was shown in [7] that for $\Delta T < 57°C$, with an error of not more than 1%, the transfer coefficients η, D, and α may be assumed to be independent of the temperature in thermodiffusion-column calculations, if the values of these coefficients are taken at the mean temperature in the column $\bar{T} = (T_1 + T_2)/2$.

Since the heat transfer is purely by conduction, the variation in temperature over the gap is linear, and the components of the acceleration of gravity, according to Fig. 1b, are $g\cos\varphi$ and $g\sin\varphi$, respectively, it follows that

$$\rho g_z = \bar{\rho} g \beta (T - \bar{T}) \cos\varphi = -\bar{\rho} g \beta \Delta T \left(\frac{1}{2} - \frac{x}{\delta}\right) \cos\varphi, \tag{4}$$

$$\rho g_y = \bar{\rho} g \beta (T - \bar{T}) \sin\varphi = -\bar{\rho} g \beta \Delta T \left(\frac{1}{2} - \frac{x}{\delta}\right) \sin\varphi. \tag{5}$$

Substituting Eqs. (4) and (5) into Eq. (2), and solving under the condition that the velocities w_x and w_y are equal to zero on the surfaces $x=\delta$ and $x=0$, we obtain

$$w_z = -\frac{\bar{\rho} g \beta \Delta T \cos\varphi}{6\eta\delta} x(x-\delta)\left(x-\frac{\delta}{2}\right), \tag{6}$$

$$w_y = -\frac{\bar{\rho} g \beta \Delta T \sin\varphi}{6\eta\delta} x(x-\delta)\left(x-\frac{\delta}{2}\right). \tag{7}$$

Equations (6) and (7) assume that the deformation of the velocity profiles due to withdrawal of material is negligibly small.

The diffusional fluxes in the column are of the form

$$j_x = -\bar{\rho} D\left[\frac{\partial c}{\partial x} - \frac{\alpha}{T} c(1-c)\frac{dT}{dx}\right], \tag{8}$$

$$j_y = \bar{\rho} w_y c - \bar{\rho} D \frac{\partial c}{\partial y}, \tag{9}$$

$$j_z = \bar{\rho} w_z c - \bar{\rho} D \frac{\partial c}{\partial z}. \tag{10}$$

The slope of the column means that, in contrast to a vertical column, in which the flux (9) is zero, this flux makes a contribution to the mass transfer.

Substituting Eqs. (8)-(10) into the continuity equation

$$\bar{\rho}\frac{\partial c}{\partial t} + \operatorname{div} \mathbf{j} = 0 \tag{11}$$

yields

$$\bar{\rho}\frac{\partial c}{\partial t} + \frac{\partial}{\partial x}\left\{-\bar{\rho} D\left[\frac{\partial c}{\partial x} - \frac{\alpha}{T} c(1-c)\frac{dT}{dx}\right]\right\} + \\ + \bar{\rho} w_y \frac{\partial c}{\partial y} - \bar{\rho} D \frac{\partial^2 c}{\partial y^2} + \bar{\rho} w_z \frac{\partial c}{\partial z} - \bar{\rho} D \frac{\partial^2 c}{\partial z^2} = 0. \tag{12}$$

Now consider an infinitesimally narrow strip dy (Fig. 1b); the quantity of a given component transferred through the cross section δdy in the z direction will be determined, i.e., according to Eq. (10)

$$d\tau_z = dy \int_0^\delta j_z dx = dy \int_0^\delta \left(\bar{\rho} w_z c - \bar{\rho} D \frac{\partial c}{\partial z}\right) dx. \tag{13}$$

Analogously, taking into account Eq. (9), the transfer in the y direction through the cross section δdz is

$$d\tau_y = dz \int_0^\delta j_y dx = dz \int_0^\delta \left(\bar{\rho} w_y c - \bar{\rho} D \frac{\partial c}{\partial y}\right) dx. \tag{14}$$

Introducing the functions

$$\Phi_1(x) = -\bar{\rho}\int_0^x w_z dx, \quad \Phi_2(x) = -\bar{\rho}\int_0^x w_y dx \tag{15}$$

the first terms in the integrand in Eqs. (13) and (14) are integrated by parts. Then, taking into account Eq. (15),

$$d\tau_z = \left[\int_0^\delta \frac{\partial c}{\partial x}\Phi_1(x)\,dx - \bar{\rho} D\int_0^\delta \frac{\partial c}{\partial z} dx + \frac{\sigma}{B} c\right] dy, \tag{16}$$

$$d\tau_y = \left[\int_0^\delta \frac{\partial c}{\partial x}\Phi_2(x)\,dx - \bar{\rho} D\int_0^\delta \frac{\partial c}{\partial y} dx\right] dz. \tag{17}$$

The derivative dc/dx may be found from Eq. (12) after integrating from 0 to x. Substituting the resulting expression into Eqs. (16) and (17) leads, after certain transformations and the usual assumptions (the basis for which is given in [8, 9]) to

$$\frac{d\tau_z}{dy} = \frac{\alpha\Delta T}{\overline{T}\delta} c(1-c) \int_0^\delta \Phi_1(x)\,dx - \frac{1}{\overline{\rho}D} \cdot \frac{\partial c}{\partial z} \int_0^\delta [\Phi_1(x)]^2\,dx -$$

$$- \frac{1}{\overline{\rho}D} \cdot \frac{\partial c}{\partial y} \int_0^\delta \Phi_1(x)\Phi_2(x)\,dx - \frac{\sigma}{\overline{\rho}DB} \cdot \frac{\partial c}{\partial z} \int_0^\delta \Phi_1(x)\,dx - \overline{\rho}D\delta \frac{\partial c}{\partial z} + \frac{\sigma}{B} c, \tag{18}$$

$$\frac{d\tau_y}{\partial z} = \frac{\alpha\Delta T}{\overline{T}\delta} c(1-c) \int_0^\delta \Phi_2(x)\,dx - \frac{1}{\overline{\rho}D} \cdot \frac{\partial c}{\partial y} \int_0^\delta [\Phi_2(x)]^2\,dx -$$

$$- \frac{1}{\overline{\rho}D} \cdot \frac{\partial c}{\partial z} \int_0^\delta \Phi_1(x)\Phi_2(x)\,dx - \frac{1}{\overline{\rho}D} \cdot \frac{\sigma}{B} \cdot \frac{\partial c}{\partial z} \int_0^\delta \Phi_2(x)\,dx - \overline{\rho}D\delta \frac{\partial c}{\partial y}. \tag{19}$$

The functions $\Phi_1(x)$ and $\Phi_2(x)$ are given by Eqs. (15), after substituting in Eqs. (6) and (7), as follows:

$$\Phi_1(x) = \frac{g\beta\overline{\rho}^2\delta\Delta T}{24\eta} x^2 \left(1 - \frac{x}{\delta}\right)^2 \cos\varphi, \tag{20}$$

$$\Phi_2(x) = \frac{g\beta\overline{\rho}^2\delta\Delta T}{24\eta} x^2 \left(1 - \frac{x}{\delta}\right)^2 \sin\varphi. \tag{21}$$

The coefficients

$$H' = \frac{\alpha\Delta T}{\overline{T}\delta} \int_0^\delta \Phi_1(x)\,dx, \quad H'' = \frac{\alpha\Delta T}{\overline{T}\delta} \int_0^\delta \Phi_2(x)\,dx,$$

$$K_c' = \frac{1}{\overline{\rho}D} \int_0^\delta [\Phi_1(x)]^2\,dx, \quad K_c'' = \frac{1}{\overline{\rho}D} \int_0^\delta \Phi_1(x)\Phi_2(x)\,dx,$$

$$K_c''' = \frac{1}{\overline{\rho}D} \int_0^\delta [\Phi_2(x)]^2\,dx, \quad K_\sigma' = \frac{\sigma}{B\overline{\rho}D} \int_0^\delta \Phi_1(x)\,dx,$$

$$K_\sigma'' = \frac{\sigma}{B\rho D} \int_0^\delta \Phi_2(x)\,dx, \quad K_d = \overline{\rho}D\delta$$

appearing in Eqs. (18) and (19) may be evaluated by replacing the functions $\Phi_1(x)$ and $\Phi_2(x)$ by their values in Eqs. (20) and (21) and calculating the integrals; we obtain

$$H' = H^0 \cos\varphi, \qquad H'' = H^0 \sin\varphi, \qquad K_c' = K_c^0 \cos^2\varphi,$$

$$K_c'' = K_c^0 \sin\varphi\cos\varphi, \qquad K_c''' = K_c^0 \sin^2\varphi,$$

$$K_\sigma' = \frac{7}{10} K_c^0 \frac{\alpha\varkappa\Delta T}{\overline{T}} \cos^2\varphi, \qquad K_\sigma'' = \frac{7}{10} K_c^0 \frac{\alpha\varkappa\Delta T}{\overline{T}} \sin\varphi, \tag{22}$$

where

$$H^0 = \frac{\alpha\overline{\rho}^2 g\beta\delta^3(\Delta T)^2}{6!\,\eta\overline{T}}, \quad K_c^0 = \frac{g^2\overline{\rho}^3\beta^2\delta^7(\Delta T)^2}{9!\,\eta^2 D}, \tag{23}$$

$$\varkappa = \frac{\sigma}{B_0 H^0}. \tag{24}$$

The parameter B_0 in Eq. (24) is the perimeter of the cylinder on which the spiral is wound and is related to the distance B between turns of the spiral by the relation

$$B = B_0 \cos\varphi, \tag{25}$$

which is obvious if a section of column of height z' is considered. It is evident that $B_0 z' = Bz$ and, on the other hand, we have

$$z' = z\cos\varphi. \tag{26}$$

Equation (25) may be derived from these two relations if it is assumed that the diameter of the wire is much less than the distance between turns of the spiral. The factor $\alpha\varkappa\Delta T/\overline{T}$ in Eq. (22) may be neglected, since it is of the order of accuracy of the transfer equation itself, as noted above.

Thus, Eqs. (18) and (19) are finally replaced by the equations

$$\frac{d\tau_z}{dy} = H^0 \cos\varphi c(1-c) - (K_c^0 \cos^2\varphi + K_d)\frac{\partial c}{\partial z} - K_c^0 \sin\varphi\cos\varphi\frac{\partial c}{\partial y} + \frac{\sigma}{B_0\cos\varphi}c, \tag{27}$$

$$\frac{d\tau_y}{dz} = H^0 \sin\varphi c(1-c) - (K_c^0 \sin^2\varphi + K_d)\frac{\partial c}{\partial y} - K_c^0 \sin\varphi\cos\varphi\frac{\partial c}{\partial z}. \tag{28}$$

The flux through unit surface in unit time is obtained by dividing the transfer into elementary areas perpendicular to the axes z and y; i.e., the fluxes are

$$j_z^* = \frac{d\tau_z}{\delta dy}, \quad j_y^* = \frac{d\tau_y}{\delta dz}. \tag{29}$$

Since mass conservation implies that

$$\bar{\rho}\frac{\partial c}{\partial t} = -\operatorname{div} \mathbf{j}^*, \tag{30}$$

Eq. (30) yields, after introducing the new variables

$$u = \frac{H^0 z}{K_c^0}, \quad v = \frac{H^0 y}{K_c^0}, \quad \theta = \frac{H^{02}t}{\bar{\rho}\delta K_c^0} \tag{31}$$

and taking into account Eqs. (27)-(29), an equation describing the change in concentration at any point of a thermodiffusion column with a spiral winding:

$$\frac{\partial c}{\partial\theta} = \cos^2\varphi(\cos^2\varphi + k)\frac{\partial^2 c}{\partial u^2} + 2\sin\varphi\cos^2\varphi\frac{\partial^2 c}{\partial u\partial v} +$$
$$+ (\sin^2\varphi + k)\frac{\partial^2 c}{\partial v^2} - \cos^2\varphi\frac{\partial}{\partial u}[c(1-c)] - \sin\varphi\frac{\partial}{\partial v}[c(1-c)] - \frac{\varkappa}{\cos\varphi}\frac{\partial c}{\partial u}, \tag{32}$$

where

$$k = \frac{K_d}{K_c^0}. \tag{33}$$

It is evident from Eq. (32) that in a column with a spiral, in contrast to a vertical column, the concentration is a function of two coordinates. In addition, whereas in a vertical liquid thermodiffusion column $K_d \ll K_c^0$ and the coefficient k in Eq. (33) may be neglected in comparison with unity, in a column with a spiral k may be commensurate with $\sin^2\varphi$ or $\cos^2\varphi$. Note also that Eq. (32) is only valid for $0 < \varphi < \pi/2$, since in these limiting cases either j_y or j_z must be set equal to zero in the physical formulation of the problem – see Eqs. (9) and (10).

Equation (32) is nonlinear and may be linearized in the cases $c(1-c) \approx c$; $c(1-c) \approx 1-c$; $c(1-c) \approx a + bc$. The conditions which must be satisfied by the solution of Eq. (32) depend on the operating conditions in the column.

Thus, a final discussion of the region of applicability of thermodiffusion columns with spiral winding will be possible after a solution to Eq. (32) has been obtained.

NOTATION

x, coordinate perpendicular to the plane of Fig. 1; y, z, z', coordinates (see Fig. 1); B, distance between turns of the spiral; L, length of the spiral; φ, inclination of spiral to the vertical; τ, mass of the given component transferred in unit time; j*, flux, i.e., transfer referred to unit cross-sectional area; u, v, θ, parameters defined in Eqs. (31); c, mass concentration; t, time; K_c^0, H^0, K_d, K_σ, $\varkappa$, see Eqs. (23)-(24); δ, gap; $\Delta T = T_1 - T_2$; T_1, T_2, temperatures of hot and cold surfaces of column; α, thermodiffusion constant; η, D, β, dynamic-viscosity, diffusion, and thermal-expansion coefficients; ρ, density.

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AN ELECTRIC ARC IN A CHANNEL BEARING A TURBULENT GAS FLOW

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A study of the positive column in an arc in a tube containing a turbulent gas flow is presented. Working formulas are given for estimating the arc parameters.

There are many papers [1-3] on the theory of arc columns stabilized by laminar gas flows; major advances have been made, as is clear from the good agreement between theory and experiment for laminar or near-laminar flow. However, the flow state may differ considerably from laminar in an electric-arc device such as a plasma source, even though the Reynolds number is small, on account of the various types of instability occurring in arcs, which cause fluctuations in the local parameters. According to current views [4], an arc becomes essentially turbulent in the presence of a moderate gas flow from the point where the positive column meets the turbulent boundary layer. The column in such a flow is described by a complicated system of equations difficult to solve. However, various simplifying assumptions can provide solutions that incorporate the major processes, which can enable one to calculate the electrical and thermal characteristics with reasonable accuracy.

1. Formulation and Solution

Here we consider an arc column in a cylindrical channel containing a turbulent gas flow; let $\rho v = \overline{\rho v} + (\rho v')$, $\rho u = (\rho u)'$, $E = \bar{E} + E'$, $S = \bar{S} + S'$, and $\sigma = \bar{\sigma} + \sigma'$, while it is assumed that the assumptions made in [1-3] apply. In the present case, we further specify that the relaxation times of the elementary processes in the plasma are small by comparison with the time scale of the turbulent pulsations. Then the positive column is described by the following equation if we assume that E is constant over the cross section of the tube and neglect turbulent heat transport along the axis, viscous dissipation, the change in the kinetic energy by comparison with the input heat, the convective transfer along the flow, and the radial energy flow arising from heat conduction and turbulence:

$$\frac{\overline{\rho v}\, h_s}{l}\, \frac{\partial \bar{S}}{\partial z} + \overline{\frac{(\rho u)'\, h_s}{R}\, \frac{\partial S'}{\partial r}} = \frac{1}{R^2 r}\, \frac{\partial}{\partial r}\left(r\, \frac{\partial \bar{S}}{\partial r}\right) + (\bar{E}^2 + \overline{E'^2})\, \sigma_s \bar{S} - \xi_s \bar{S}, \tag{1}$$

$$\langle I \rangle = 2\pi R^2 \sigma_s \langle E \rangle \int_0^1 \bar{S} r dr \tag{2}$$

subject to the conditions

$$\bar{S}(r,\ 0) = \varphi(r),\quad \bar{S}(1,\ z) = 0,\quad \frac{\partial \bar{S}}{\partial r}\ (0,\ z) = 0,\quad \overline{\rho v} = \text{const.} \tag{3}$$

Estimation of the terms in the equations for the arc shows that this model applies for reasonably long channels provided that the discussion is restricted to the region of developed turbulence for low Mach numbers. The instantaneous current and voltage are variable, so Ohm's law in (2) has been written for the effective values. The turbulent pulsations are completely random, so the phase difference between I and E may be taken as zero.

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